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**INFLATION-LINKED BONDS, NOMINAL BONDS, AND
COUNTERCYCLICAL MONETARY POLICIES**

By

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Inflation-Linked Bonds, Nominal Bonds, and Countercyclical Monetary Policies

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Abstract:

Although inflation-linked bonds have many advantages, nominal bonds are the most important instrument to finance public debts throughout the world. One explanation that the literature has offered is that nominal bonds make countercyclical monetary policies more effective. This paper reconsiders this argument with a model that features an inflation risk premium in the nominal bonds interest rate. In this model, nominal bonds help to stabilize the economy, but also add to debt service costs. The paper finds that the debt service costs channel is very powerful: in the case of discretionary policy-making, inflation-linked bonds always outperform nominal bonds. The case of commitment qualifies this result. Still, also commitment cannot explain the occurrence of large stocks of nominal bonds alongside small stocks of inflation-linked bonds.

Keywords:

Inflation-indexed Bonds, Time-Consistent Discretionary Monetary Policies, Stabilization Properties of Monetary Policies, Inflation Risk Premium

JEL Codes:

E52, H63

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1 Introduction

Inflation-linked bonds are becoming increasingly popular. Since the UK and the US launched inflation-linked bonds in 1981 and 1997 respectively, more and more countries have started issuing inflation-indexed bonds, among which are countries as Germany, France, Italy and Spain. In addition, the amounts outstanding are increasing. The universal market for inflation-linked bonds has grown from below \$500 billion at the start of the century to more than \$3.000 billion in 2016 (PIMCO, 2020).

Yet, even today the majority of public debt is financed through nominal bonds. Westerhout and Ciocyte (2017) report that the fraction of the public debt that is financed with inflation-linked bonds is sizeable in the UK, 28 percent, but much smaller in other countries: 14 percent in France, 11 percent in the US and 10 percent in Italy. Furthermore, the number of countries that do not issue inflation-indexed bonds at all is still pretty large.

This picture points to a puzzle that has been recognized for a long time but still has not been solved: why are nominal bonds rather than inflation-linked bonds the major type of debt? For there are several factors that call for using inflation-linked debt. A fundamental argument is that inflation-linked bonds generate real returns that, unlike those of nominal bonds, do not vary with inflation.¹ This should make inflation-linked bonds more attractive to both their holders and their issuers. Furthermore, due to the protection of investors from inflation risk, the interest rate on inflation-linked bonds may be below that on nominal bonds, allowing governments in turn to reduce their average debt service costs (Fischer, 1975; Campbell and Shiller, 1996; Campbell and Viceira, 2001). Using tax smoothing as objective, optimal debt management calls for the use of long-term inflation-linked bonds (Barro, 2003). As stressed in the time consistency literature, inflation-linked bonds hold the promise of lower inflation and lower associated welfare costs (Calvo, 1978; Lucas and Stokey, 1983; Barro and Gordon, 1983a, 1983b). Inflation-linked bonds can also be used to derive inflation expectations, which is helpful for the conduct of monetary policies (Campbell *et al.*, 2009).

A disadvantage of inflation-linked bonds is that their markets may be

¹In practice, nominal returns do not vary 1-to-1 with inflation. Payments are often linked to lagged inflation rates. In the case of the UK, these are the inflation rates from 3 to 8 months before (UK Debt Management Office, 2020). Next, many ILB-issuing countries offer deflation floors at maturity (PIMCO, 2020). Further, the price index that is relevant for some investor will generally not coincide perfectly with the price index used for indexing the inflation-linked bond.

less liquid than those of nominal bonds (Campbell *et al.*, 2009; Gürkaynak *et al.*, 2010; Fleckenstein *et al.*, 2014). This relative lack of liquidity has been found to be particularly important when the bonds are introduced and in times of financial distress such as in 2008 (Campbell *et al.*, 2009). Furthermore, as shown by Bohn (1988) and Calvo and Guidotti (1990), nominal bonds allow the government to hedge against shocks to its budget if adverse shocks to the government budget correlate with an increase in inflation. Hence, nominal debt can reduce the variability of distortionary tax rates across states of nature. Further, part of government spending may be nominal due to nominal contracts, such as wage contracts. This may offer a third reason to finance the public debt partly with nominal bonds.

This paper asks whether the dominance of nominal bonds can be explained from their attractive stabilization properties. Therefore, I construct a model in the spirit of the Barro-Gordon model of discretionary monetary policies (Barro and Gordon, 1983a; 1983b; Rogoff, 1985; Alesina and Grilli, 1992; Walsh, 1995) and add to this a portfolio decision. In this model, like in the Barro-Gordon model, monetary policies stabilize the economy and imply an inflation bias. However, my model adopts a different mechanism in linking surprise inflation to an expansion of the economy. This mechanism relies fully on nominal bonds, thereby giving maximum potential for nominal bonds to stabilize the economy. In addition and similar to Walsh (1995), the model allows the central banker to be imperfectly informed about the supply shocks she wants to fight.

In the model, households decide on the shares of nominal bonds and inflation-linked bonds in their portfolios before an output shock has materialized and monetary policies are carried out. As will be shown below, the monetary policies in this model imply that consumption and inflation are negatively correlated. Hence, households command a positive inflation risk premium on nominal bonds in order to compensate for their unattractive hedging properties. This risk premium will turn out to be crucial for the main results in this paper and is a major difference with a large literature that assumes risk neutrality (see Bohn (1988, 1990) and Calvo and Guidotti (1990, 1993)).

In order to understand what drives the optimal financing structure of the public debt, this paper distinguishes between the direct effects of nominal debt upon household welfare and the indirect effects, *i.e.* effects that run through debt service costs. The direct effects are quite intuitive and similar to those obtained by Calvo and Guidotti (1990). For a sufficiently high output gap (a sufficiently low variance of supply shocks), optimal public debt policies use only inflation-indexed bonds, whereas in the case of a

lower output gap (higher variance), the optimal amount of nominal debt is strictly positive, decreasing in the output gap and increasing in the variance of supply shocks.

However, in order to get the complete picture, we must add the indirect effects and these indirect effects work against nominal bonds. Due to the inflation risk premium that distinguishes nominal bonds from inflation-linked bonds, the introduction of nominal bonds implies an increase in debt service costs and thus higher taxes, lower structural output and higher inflation. These indirect effects turn out to dominate the direct effects. Hence, it is optimal not to issue any nominal bonds at all. This result holds true irrespective the size of the output gap, the variance of supply shocks or the degree of informedness of the central banker.

This result does change however, if we replace the assumption of discretionary policies with a form of commitment. Basically, in the case of commitment, there is no inflation bias and the effect that nominal bonds exert upon this inflation, directly and indirectly, disappears. This in itself favors nominal bonds: in the case of commitment, it is optimal to have either zero nominal bonds or zero inflation-linked bonds. This result also holds true in a more general case, a case that nests the cases of discretion and commitment. This underlines that inclusion of an inflation risk premium alone may not be sufficient to find that inflation-linked bonds dominate nominal bonds: there need to be a sufficiently large effect of the output gap upon average inflation. Still, commitment, like discretion, cannot explain the observation that ILB-issuing countries use both nominal and inflation-linked bonds as financing tools.

This paper draws on a large literature on discretionary monetary policies (Calvo, 1978; Lucas and Stokey, 1983; Barro and Gordon, 1983a; 1983b; Rogoff, 1985; Alesina and Grilli, 1992; Walsh, 1995). The paper also relates to a literature that focuses on the tax smoothing properties of different types of debt (Barro, 2003; Bohn, 1990). Furthermore, the paper connects to a literature that studies the interaction between the tax smoothing properties of economic policies and their implications for the inflation bias of monetary policies (Bohn, 1988; Calvo and Guidotti, 1990; 1993; Athey *et al.*, 2005; Martin, 2011).

There is a growing literature that compares the virtues of the two types of bonds. Campbell and Viceira (2001) compare inflation-indexed bonds and nominal bonds from the perspective of long-term, conservative investors. They find that inflation-linked bonds are more attractive to investors and that the associated welfare gains can be large. Diaz-Gimenez *et al.* (2008) compares nominal debt with price-indexed debt in a cash-in-advance model.

They find that if the intertemporal elasticity of substitution equals one, indexed debt dominates nominal debt. If this elasticity differs from one, nominal debt can outperform indexed debt, however. Alfaro and Kanczuk (2010) adopt a quantitative calibrated model to assess the importance of the inflation bias and the hedging gains that are connected with nominal bonds. They find that the optimal amount of nominal debt is zero, both for the US economy and that of Brazil, countries that have very different fractions of inflation-linked debt. Hatcher (2014) compares the two types of debt under two regimes for monetary policies, inflation targeting and price-level targeting. Under inflation targeting, indexed debt is dominant, whereas nominal debt can outperform indexed debt under price-level targeting. However, abstracting from indexation lags, indexed debt is superior also under price-level targeting. Westerhout and Beetsma (2019) compare inflation-linked bonds and nominal bonds under different fiscal regimes. They show that inflation-linked bonds may stabilize fiscal indicators such as the public deficit and public debt ratio, which may be worthwhile if there exist bounds for these ratios, such as under the Stability and Growth Pact.

The paper is structured as follows. Section 2 sets up the central banker's model of discretionary monetary policies. Section 3 uses this model to elaborate its implications for the inflation risk premium and, related, the level of structural output. Next, optimal debt policies are explored in section 4. Section 5 considers two alternatives to the concept of discretionary monetary policies. Finally, section 6 contains concluding remarks.

2 A model of countercyclical monetary policies

We adopt a model in the spirit of the Barro-Gordon model of discretionary monetary policies (Barro and Gordon, 1983a; 1983b; Rogoff, 1985; Alesina and Grilli, 1992; Walsh, 1995). In this model, a central banker chooses the rate of inflation such as to minimize a quadratic loss function which has two arguments: consumption and inflation. This loss function represents the preferences of consumers and it is this loss function that consumers try to minimize when they choose their portfolio of nominal and inflation-linked bonds. This portfolio behaviour will be discussed in the next section.

In choosing the rate of inflation, the central banker takes into account the relation between consumption and surprise inflation. This relation is based on three elements. First, the government in our model chooses the tax rate on labour such as to equate tax revenues to debt service costs. Second, firms are profit maximizers and base their labour demand on the prevailing tax

rate on labour. Third, households spend what they earn. Hence, unexpected inflation produces higher consumption: unexpected inflation induces the government to lower the tax rate on labour income, induces firms to attract more labour as wage costs decrease, and induces households to consume the increase in output.

One might think that the model in this paper assumes cooperation between monetary and fiscal policymakers. This is not a good interpretation of the model, however. The model does make an assumption on the fiscal authority, namely that it pursues balanced budget policies by changing the employer tax to absorb variations in debt service costs. This does not necessarily imply cooperation, however. Its only assumption is that the monetary policymaker, when deciding about optimal monetary policies, accounts for the reaction by fiscal authorities to any surprise inflation it produces. Such behaviour can reflect cooperation; it can also reflect rational decision-making by a monetary policymaker in a world in which the fiscal authority lives according to a well-specified rule.

The economy we study is small relative to the world economy. Both the interest rate on inflation-linked bonds and the wage rate are determined on world markets and thus exogenous. Note that the interest rate on nominal bonds is endogenous, however, reflecting the inflation risk premium that relates to among other things the amount of nominal bonds. The model abstracts from any dynamic considerations: both the public debt and the stock of household wealth are taken as given. This permits us to derive analytical solutions.

Our model distinguishes households, firms, the government and the central banker. We start the discussion with households.

2.1 Households

Consumption of households equals the sum of labour income, the profits of firms and the returns on the bonds that households hold,

$$c = wl + \pi + s \tag{1}$$

where w denotes the wage rate, l denotes employment, π denotes profits and s denotes bond returns.

Profits of firms are defined as revenues minus costs, with the latter consisting of wage costs plus a tax levied on the labour costs of firms,

$$\pi = y - (w + \tau)l \tag{2}$$

where y stands for output and τ denotes the wage tax per unit of labour employed.

Upon using t to refer to the revenues from taxing labour income, *i.e.* $t \equiv \tau l$ and combining equations (1) and (2), we derive the following expression for consumption:

$$c = y - t + s \quad (3)$$

Bond returns consist of the returns on nominal bonds and those on inflation-linked bonds. We will elaborate this below when we discuss the role of the government.

Output is determined in the model of firms, which we will describe now.

2.2 Firms

The production of firms is described with a production function which has decreasing returns to labour:

$$y = l^\eta + \mu \quad 0 < \eta < 1 \quad (4)$$

Here, μ denotes an error term, which we assume to be normally distributed with zero mean and variance $0 < \text{Var}(\mu) < \infty$.²

In maximizing profits, the firm takes as given the price of labour. This consists of the wage rate w , determined on the world market for labour, and τ , the tax levied on the labour costs of firms. Profit maximization then implies the following expression for the demand for labour:

$$l = \left(\frac{w + \tau}{\eta} \right)^{\frac{1}{\eta-1}} \quad (5)$$

Inserting this expression into the production function gives us an expression for output as a function of the tax rate on labour. For convenience, we linearize this expression. For the same reason, we elaborate a linearized expression for tax revenues as a function of the tax rate. Combining the two, we can derive the following expression for output as a function of tax revenues (see appendix A for the full derivation):

$$y = y_n - \epsilon t + \mu \quad (6)$$

²In order to avoid negative output, the distribution of μ must be bounded from below, which a normal distribution is not. We abstract from this inconsistency, as it will be irrelevant here in any practical sense.

As we will see below, tax revenues relate to monetary policies. y_n is exogenous and can be interpreted as natural output.

Given this equation for output, we can rewrite the equation for consumption, (3), as

$$c = y_n - (1 + \epsilon)t + s + \mu \quad (7)$$

The consumption equation can be further elaborated by invoking the government budget constraint. Hence, we now turn to the model of the government.

2.3 The government

Capital income, s , adds the real returns on nominal bonds to those on inflation-linked bonds:

$$s = (i_N - \tilde{p})b_N + r_R b_R \quad (8)$$

As regards interest rates, we adopt i to refer to nominal rates and r to refer to real rates. Subscript N denotes nominal bonds and subscript R inflation-linked bonds. Hence, i_N is the nominal rate of return on nominal bonds and r_R the real rate of return on inflation-linked bonds. The former is endogenous, the second is exogenous (determined on the world capital market). b_N and b_R denote the stocks of nominal and inflation-linked bonds, which add up to the total public debt, b , which we assume strictly positive: $b \equiv b_N + b_R > 0$. We assume both types of bonds are issued in non-negative amounts: $0 \leq b_N, b_R \leq b$.

We define the expected real rate of interest on nominal bonds as the corresponding nominal interest rate minus the expected rate of inflation:

$$E(r_N) = i_N - E(\tilde{p}) \quad (9)$$

Combining equations (8) and (9), we can write capital income as a function of three terms,

$$s = r_R b + P b_N - (\tilde{p} - E(\tilde{p}))b_N \quad (10)$$

where P is defined as $E(r_N) - r_R$, the inflation risk premium on nominal bonds.

The first term on the RHS of equation (10) denotes the debt service that would apply if all debt was financed with inflation-linked bonds. The second term denotes the additional debt service that is due to the inflation

risk premium. The last term on the RHS of equation (10) gives the change in the (real) debt service on nominal bonds that is due to unexpected inflation, $\tilde{p} - E(\tilde{p})$.

Tax revenues t follow from the government budget constraint which equates those revenues to the debt service on nominal and inflation-linked bonds:

$$t = s \quad (11)$$

Combining this result with the equation for consumption, (7), we derive the following version of the consumption equation:

$$c = y'_n + \epsilon b_N(\tilde{p} - E(\tilde{p})) + \mu \quad (12)$$

with y'_n defined as $y_n - \epsilon(r_R b + P b_N)$.

y'_n will be denoted as structural output, to distinguish it from natural output, y_n . The two concepts share that they are unrelated to counter-cyclical monetary policies. They differ on account of public debt policies. In particular, structural output is affected by public debt policies, whereas natural output is unrelated to debt policies.

Equation (12) shares with the Lucas supply function (Lucas, 1973) and the wage-contract version of the supply function (Rogoff, 1985) that surprise inflation causes the economy to expand. According to the Lucas supply function, surprise inflation induces firms to expand output as they interpret nominal price changes as relative changes. According to the wage contract theory, output expands as surprise inflation lowers real wages. In our model, surprise inflation generates capital gains on nominal bonds which the government uses to lower employer taxes and to which firms respond by expanding employment and output. Note that the mechanism relies exclusively on nominal bonds: with zero nominal bonds, surprise inflation has no output effect.³

³The mechanism embedded in equation (12) is not the only way to model an effect from surprise inflation upon consumption that gives a role to nominal debt. For example, we could have included a labour income tax in our model, to be paid by employees, assuming that the before-tax wage rate is determined on international markets. The reduction in debt service costs that is due to surprise inflation would then bring about a lower tax on labour income and an increase in labour supply and output, assumed that the uncompensated labour supply elasticity is positive. In general, any government policy that uses freed debt service costs to raise the supply of output could replace our mechanism. Note that not all types of policies meet these conditions. Using freed debt service costs to raise government spending on goods and services would not raise output. Higher public spending would simply replace lower private spending, due to reduced capital income from the holding of bonds.

The relation between consumption and unexpected inflation in equation (12) is taken into account by the central banker when choosing the rate of inflation. To see what this implies for inflation, we will now describe the behaviour of the central banker.

2.4 The central banker

The central banker chooses the inflation rate such as to minimize the following household loss function:

$$L = \frac{1}{2}\tilde{p}^2 + \frac{1}{2}\delta(c - \hat{c})^2 \quad (13)$$

This loss function is standard in the literature on discretionary monetary policies. It assumes that households dislike inflation, \tilde{p} , for reasons that we do not further specify. Similarly, the household dislikes consumption below a certain target level, \hat{c} . Also standard is that we assume $\hat{c} > E(c)$ or, equivalently, $\hat{c} > y'_n$ (see equation (12)), reflecting some existing distortion in the economy (Barro and Gordon, 1983a). As y'_n is endogenous in our model, we can at this point not be very precise about what $\hat{c} > y'_n$ implies. We will come back to this below when we have obtained a reduced-form expression for structural output.

The central banker now chooses that rate of price inflation that minimizes the loss function (13), taking into account the relation between consumption and unexpected inflation (12). She does so after a supply shock has realized. The central banker is generally imperfectly informed about this shock, however.

In particular, we assume that the central banker receives a signal, to be denoted as μ_p , which equals the shock plus some measurement error variable: $\mu_p \equiv \mu + \eta$. As regards η , we assume that it follows a normal distribution with zero mean and variance $0 \leq \text{Var}(\eta) < \infty$ and that it is uncorrelated with the supply shock μ . Hence, the expectation of μ , conditional upon observing the signal μ_p , equals $\theta\mu_p$, where θ is the ratio of the variance of the shock and the variance of the signal: $\theta \equiv \text{Var}(\mu)/(\text{Var}(\mu) + \text{Var}(\eta))$ (Walsh, 1995). Given our assumptions on $\text{Var}(\mu)$ and $\text{Var}(\eta)$, θ is strictly positive and equals one in the polar case of a perfectly informed central banker.

The solution to the problem of the central banker implies the following expressions for inflation and consumption:

$$\tilde{p} = \delta\epsilon b_N(\hat{c} - y'_n) - \left(\frac{\theta\delta\epsilon b_N}{1 + \delta(\epsilon b_N)^2} \right) (\mu + \eta) \quad (14)$$

$$c = y'_n + \left(\frac{1 + (1 - \theta)\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \mu - \left(\frac{\theta\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \eta \quad (15)$$

These expressions have a few standard properties.⁴ First, a higher output gap, defined as $\hat{c} - y'_n$, implies higher inflation, but does not affect consumption (recall that $\hat{c} > y'_n$). As long as the output target is public information, it will be incorporated in inflationary expectations and consumption will remain unaffected.

Second, supply shocks induce negative correlation between inflation and consumption as monetary authorities use inflation to stabilize the economy. Observation errors, on the contrary, imply positive correlation between consumption and inflation. The reason is that in case of an error, output changes only on account of the surprise inflation that the policymaker produces.

Equations (14) and (15) also indicate how imperfect information affects the behaviour of the central banker. More noise in the signals that the central banker receives about output shocks, *i.e.* a higher variance of observation errors $Var(\eta)$, implies a lower value for θ . As the central banker reacts to the expected value of the supply shock, which is given by $\theta\mu_p$, more noise makes the central banker less active. Hence, inflation becomes less responsive to both output shocks and observation errors. This magnifies the impact upon consumption of supply shocks and reduces that of observation errors.

Importantly, nominal debt also affects the responsiveness of inflation to an output shock. If we differentiate the coefficient of $(\mu + \eta)$ in equation (14), $\theta\delta\epsilon b_N/(1 + \delta(\epsilon b_N)^2)$, with respect to b_N , we find that the coefficient is increasing in b_N if $0 < b_N < 1/(\sqrt{\delta\epsilon})$ and decreasing in b_N if $b_N > 1/(\sqrt{\delta\epsilon})$. This reflects that two forces work against each other. A higher amount of nominal debt makes stabilization policies more effective. This induces the central banker to make more use of the inflation instrument, but at the same time makes it less necessary to use this instrument. For low levels of nominal debt, the former argument dominates, for higher levels of nominal debt the latter one.

The derived expressions for the rate of inflation and consumption, (14) and (15), can be used to obtain the following expressions for their means, their variances and their covariance:

$$E(\tilde{p}) = \delta\epsilon b_N(\hat{c} - y'_n) \quad (16)$$

⁴Equations (14) and (15) generalize those derived in Alesina and Grilli (1992). They are obtained if we add to their model a non-zero level of natural output and imperfect information on the part of the central banker and further allow the coefficient of the Phillips curve to deviate from one.

$$E(c) = y_n' \quad (17)$$

$$Var(\tilde{p}) = \left(\frac{\delta \epsilon b_N}{1 + \delta(\epsilon b_N)^2} \right)^2 \theta Var(\mu) \quad (18)$$

$$Var(c) = \left(\frac{1 + 2(1 - \theta)\delta(\epsilon b_N)^2 + (1 - \theta)\delta^2(\epsilon b_N)^4}{(1 + \delta(\epsilon b_N)^2)^2} \right) Var(\mu) \quad (19)$$

$$Cov(c, \tilde{p}) = \frac{-\theta \delta \epsilon b_N}{(1 + \delta(\epsilon b_N)^2)^2} Var(\mu) \quad (20)$$

Equations (18) to (20) are expressed in terms of θ and $Var(\mu)$ rather than $Var(\mu)$ and $Var(\eta)$ (see appendix B for the derivation of these equations). This allows to see clearly the role of θ , which measures the information on part of the central banker. In particular, the less informed the central banker (the lower is θ), the lower is the variance of inflation, the higher is the variance of consumption and the less negative is the covariance between consumption and inflation.

Importantly, this covariance is always non-positive (see equation (20)). One might reason that a larger occurrence of observation errors (a higher value for $Var(\eta)$) would increase the weight of positively correlated changes in consumption and inflation, turning the covariance positive. A larger occurrence of observation errors also makes the central banker less informed, however, and less inclined to change the rate of inflation. The latter mechanism reduces the covariance in an absolute sense and is sufficiently strong to prevent the covariance from becoming positive. This feature also explains that the assumption of imperfect information on part of the central banker qualifies our results, but does not fundamentally change them.

One might argue that the non-positive nature of the covariance is overly restrictive. This argument misses the question this paper wants to answer, however. That is, can we explain the dominance of nominal bonds in the real world from their attractive stabilization properties? If we constructed the model such that observation errors would dominate supply shocks and the covariance between consumption and inflation were positive, nominal debt would destabilize the economy rather than stabilizing it.

Equations (16) to (20) also point to the role of nominal debt. In particular, equation (16) shows that nominal debt increases expected inflation. This reflects the well-known inflation bias. Next, equation (19) can be used to derive that consumption variability is a negative function of nominal debt.⁵ The reason is that nominal debt flattens the Phillips curve, thereby

⁵Falchetti and Missale (2002) present some empirical evidence for the model prediction that the variance of output is decreasing in the amount of nominal public debt.

increasing the effectiveness of monetary stabilization policies.

These two factors suggest that the optimal amount of nominal debt will reflect a trade-off between the inflation bias and consumption variability. However, note that structural output enters equations (16) and (17). Below, I will show that structural output relates to public debt policies. This indicates it is too early at this stage to conclude something about the welfare properties of nominal debt.

3 The inflation risk premium and structural output

As indicated above, households in our model allocate a given amount of financial wealth over two assets, namely nominal bonds and inflation-linked bonds. How do they solve this allocation problem? Given that they must choose their holdings of the two types of bonds before any shock has materialized, they minimize the expected value of the utility loss function in equation (13):

$$E(L) = E\left(\frac{1}{2}\tilde{p}^2 + \frac{1}{2}\delta(c - \hat{c})^2\right) \quad (21)$$

The relation between consumption and the household portfolio is described by the consumption function (equation (7)), where we use equation (8) to substitute for bond returns:

$$c = y_n - (1 + \epsilon)t + (i_N - \tilde{p})b_N + r_R b_R + \mu \quad (22)$$

The amount of nominal bonds that is optimal for the household then obeys the first-order condition $\partial E(L)/\partial b_N = 0$, which can be written as an expression for the inflation risk premium:

$$P = -\frac{Cov(c, \tilde{p})}{(\hat{c} - y'_n)} \quad (23)$$

Equation (23) expresses that the inflation risk premium is proportional with the covariance between consumption and inflation. In the previous section we have derived that this covariance is non-positive (the covariance is zero only if all debt is inflation-indexed ($b_N = 0$)). Given the assumption $\hat{c} > y'_n$, equation (23) then indicates that the risk premium is non-negative (and zero only if all debt is inflation-indexed). The interpretation of the inflation risk premium is that of a compensation for inflation risk. Indeed, in case of a

negative correlation between inflation and consumption, nominal bonds are a bad hedge against consumption shocks and a premium is needed to make households willing to hold them.

It is worthwhile to pause here a little and to ask whether the non-negativeness of the inflation risk premium corresponds with findings in the empirical literature. In general, we can say that empirical estimates of the inflation risk premium vary a lot and are not always positive. In an overview paper, Kupfer (2018) finds that, overall, the estimates of the inflation risk premium are more positive than negative, however.⁶ Similarly, the estimates in Bekaert *et al.* (2010) are, except one, all positive (and increasing in the horizon). Furthermore, d'Amico *et al.* (2018), reviewing that part of the literature that bases estimates of the inflation risk premium on the covariance between inflation and the marginal utility of wealth, finds that the estimated inflation risk premium is positive, ranging between 10 and 100 basis points.

In order to find a reduced-form expression for the inflation risk premium, we now use the derived expression for the covariance between consumption and inflation, equation (20), and the expression for structural output, $y'_n = y_n - \epsilon(r_R b + P b_N)$. This leads to the following quadratic equation for the inflation risk premium:

$$(\epsilon b_N)P^2 + (\hat{c} - y_n + \epsilon r_R b)P - \frac{\delta \epsilon b_N}{(1 + \delta(\epsilon b_N)^2)^2} \theta \text{Var}(\mu) = 0 \quad (24)$$

Before choosing which of the two candidate solutions applies, note that $x > 0$, where x is a shortcut for $\hat{c} - y_n + \epsilon r_R b$, which is exogenous. $x > 0$ follows from writing x as $\hat{c} - y'_n - \epsilon P b_N$ and noting that $\hat{c} > y'_n$ holds true (by assumption) for any value of b_N , including the zero value. For $x > 0$, the only candidate solution that implies $P \geq 0$ and $P = 0$ if $b_N = 0$ reads as follows:

$$P = \frac{1}{(2\epsilon b_N)} \left[\sqrt{x^2 + \frac{4\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \theta \text{Var}(\mu)} - x \right] \quad (25)$$

Equation (25) shows that, if $b_N > 0$, the risk premium is increasing in the variance of supply shocks, $\text{Var}(\mu)$, decreasing in the variance of information errors, $\text{Var}(\eta)$, and decreasing in the consumption target, \hat{c} . The risk

⁶Note that Kupfer (2018) finds that estimates of the inflation risk premium vary a lot over time and that they are negative in some year in many analyses. This does not seem related to correction for liquidity factors, suggesting therefore that shocks that imply positive correlation between inflation and output do play a role.

premium is also a function of the amount of nominal debt. In order to learn about the shape of this function, it is more instructive to elaborate an expression for debt service costs, $r_R b + P b_N$.

Using the expression in equation (25), we can derive the following expression for debt service costs:

$$r_R b + P b_N = r_R b + \frac{1}{2\epsilon} \left[\sqrt{x^2 + \frac{4\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \theta \text{Var}(\mu)} - x \right] \quad (26)$$

Differentiation with respect to nominal debt gives the following result:

$$\frac{d(r_R b + P b_N)}{db_N} = \frac{2\theta \text{Var}(\mu) \delta \epsilon b_N (1 - \delta(\epsilon b_N)^2)}{(1 + \delta(\epsilon b_N)^2)^3 \sqrt{x^2 + \frac{4\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \theta \text{Var}(\mu)}} \quad (27)$$

This expression makes clear that debt service costs are a hump-shaped function of nominal debt: increasing in nominal debt if $0 < b_N < 1/(\sqrt{\delta}\epsilon)$ and decreasing if $b_N > 1/(\sqrt{\delta}\epsilon)$. Note that we observed the same pattern before, namely when discussing the expression for price inflation (equation (14)). For low levels of nominal debt, more nominal debt implies more inflation for a given output shock as nominal debt makes monetary policies more effective. For high levels of nominal debt, the opposite holds true: more nominal debt implies less inflation for a given output shock as less inflation is needed to achieve output stabilization. It is easy to see that the two are connected. Indeed, the hump-shaped pattern that we derived for inflation also applies to the covariance between inflation and consumption, the inflation risk premium and debt service costs.

Combining the reduced-form expression for debt service costs, equation (26), with the expression for structural output, $y'_n = y_n - \epsilon(r_R b + P b_N)$, yields a reduced-form expression for the latter:

$$y'_n = \hat{c} - \frac{1}{2} \left[\sqrt{x^2 + \frac{4\theta\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \text{Var}(\mu)} + x \right] \quad (28)$$

Recall that, in setting up the model for the central banker, we assumed that the target level of consumption exceeds structural output, *i.e.* $\hat{c} > y'_n$. We can now use the result that $x > 0$ to see what this assumption exactly means. That is that $\hat{c} > y_n - \epsilon r_R b$.

Now that we have derived a reduced-form expression for structural output, we can derive the optimal composition of the public debt. These optimal debt policies follow from minimizing the social welfare loss function with b_N as instrument.

4 Optimal debt policies

To find the optimal composition of the public debt, we adopt again the ex ante value of the loss function (equation (21)). However, we rewrite it here as a function of the means and variances of inflation and consumption:

$$E(L) = \frac{1}{2}E(\tilde{p})^2 + \frac{1}{2}Var(\tilde{p}) + \frac{1}{2}\delta(E(c) - \hat{c})^2 + \frac{1}{2}\delta Var(c) \quad (29)$$

Inserting into equation (29) the expressions for the means and variances of inflation and consumption (equations (16) to (19)) gives the following expression:

$$\begin{aligned} E(L) &= \frac{1}{2}\delta(1 + \delta(\epsilon b_N)^2)(\hat{c} - y'_n)^2 \\ &+ \frac{1}{2}\delta \left(\frac{1 + (1 - \theta)\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) Var(\mu) \end{aligned} \quad (30)$$

This expression for the welfare loss of discretionary monetary policies reflects the trade-off between excessively high inflation and output stabilization. The first term on the RHS of equation (30) reflects that discretionary monetary policies produce too much inflation, thereby lowering social welfare. The second term reflects that these policies stabilize the economy and thus increase social welfare (without these policies, the second term at the RHS of equation (30) would have read as $1/2\delta Var(\mu)$).

The optimal amount of nominal debt is determined by direct and indirect effects. Direct effects occur irrespective debt service costs. Indirect effects occur through changes in debt service costs. Although the two can strictly speaking not be separated, it is instructive to do so. We first focus on the direct effects.

4.1 Direct effects

In order to exclude the indirect effects, we take y'_n as exogenous in this subsection. We can then, given this condition, write the derivative of the welfare loss with respect to the amount of nominal debt as the sum of two effects:

$$\frac{\partial E(L)}{\partial b_N} = (\delta\epsilon)^2 b_N \left[(\hat{c} - y'_n)^2 - \frac{\theta Var(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \right] \quad (31)$$

The first-order condition that gives us the optimal amount of nominal debt can be written as $\partial E(L)/\partial b_N = 0$. In general, this first-order condition has

three candidate solutions for b_N . In each of the two cases we will consider, two of the three can be ruled out, however.

The first case to consider is where $(\hat{c} - y'_n) \geq \sqrt{\theta Var(\mu)}$. In this case, two of the three solutions are complex. The only real solution obeys the second-order condition. This solution reads as follows,

$$b_N^* = 0 \quad (32)$$

where the asterisk is used to denote optimal debt policies.

In the second case to consider, $(\hat{c} - y'_n) < \sqrt{\theta Var(\mu)}$. In this case, all three candidate solutions are real, but only one of them obeys the second-order condition and lies within the admissible domain $(0, b)$. This solution reads as follows:

$$b_N^* = \min \left[\frac{1}{\epsilon} \sqrt{\frac{1}{\delta} \left(\frac{\sqrt{\theta Var(\mu)}}{(\hat{c} - y'_n)} - 1 \right)}, b \right] \quad (33)$$

We summarize these results in the form of two propositions.

Proposition 1a:

If the output gap is sufficiently large, *i.e.* $(\hat{c} - y'_n) \geq \sqrt{\theta Var(\mu)}$, the optimal financing structure of the public debt features zero nominal debt.

Proposition 1b:

If the output gap is sufficiently small, *i.e.* $(\hat{c} - y'_n) < \sqrt{\theta Var(\mu)}$, the optimal financing structure of the public debt features a strictly positive amount of nominal debt, as specified in equation (33).

Figure 1 illustrates these propositions by adopting a high and low value for the consumption target \hat{c} , namely 10.0 and 6.0 respectively.⁷ From propositions 1a and 1b, it is not immediately clear how high and how low \hat{c} should be, as y'_n is endogenous. The high value for \hat{c} obeys $x \geq \sqrt{\theta Var(\mu)}$, which appendix C shows is a sufficient condition for $(\hat{c} - y'_n) \geq \sqrt{\theta Var(\mu)}$. Similarly, the low value for \hat{c} obeys $x < 3/4 \sqrt{\theta Var(\mu)}$, which the same appendix shows is a sufficient condition for $(\hat{c} - y'_n) < \sqrt{\theta Var(\mu)}$.

The upper panel of figure 1 displays the two components of $E(L)$ *i.e.* the two terms at the RHS of equation (30), for $\hat{c} = 10$. The line 'output gap' refers to the first of these terms. The figure shows that the welfare

⁷The numerical calculations that are used to produce Figures 1 to 4 adopt the following parameter values: $\delta=0.1$, $\epsilon=0.5$, $y_n=4.0$, $\theta=1.0$, $Var(\mu)=10.0$, $r_R=0.01$, and $b=10.0$.

loss due to the output gap is increasing in b_N for all $b_N > 0$. Similarly, the line 'variability' refers to the second of the two terms at the RHS of equation (30). This stabilization term is decreasing in b_N for all $b_N > 0$: more nominal debt reduces the welfare loss through its effect upon output and price stability. The line 'total' adds the two effects. It is increasing for all $b_N > 0$. Hence, in the case of a relatively high consumption target ($\hat{c} = 10$), the amount of nominal debt that produces the lowest welfare loss is zero, as stated in proposition 1a.

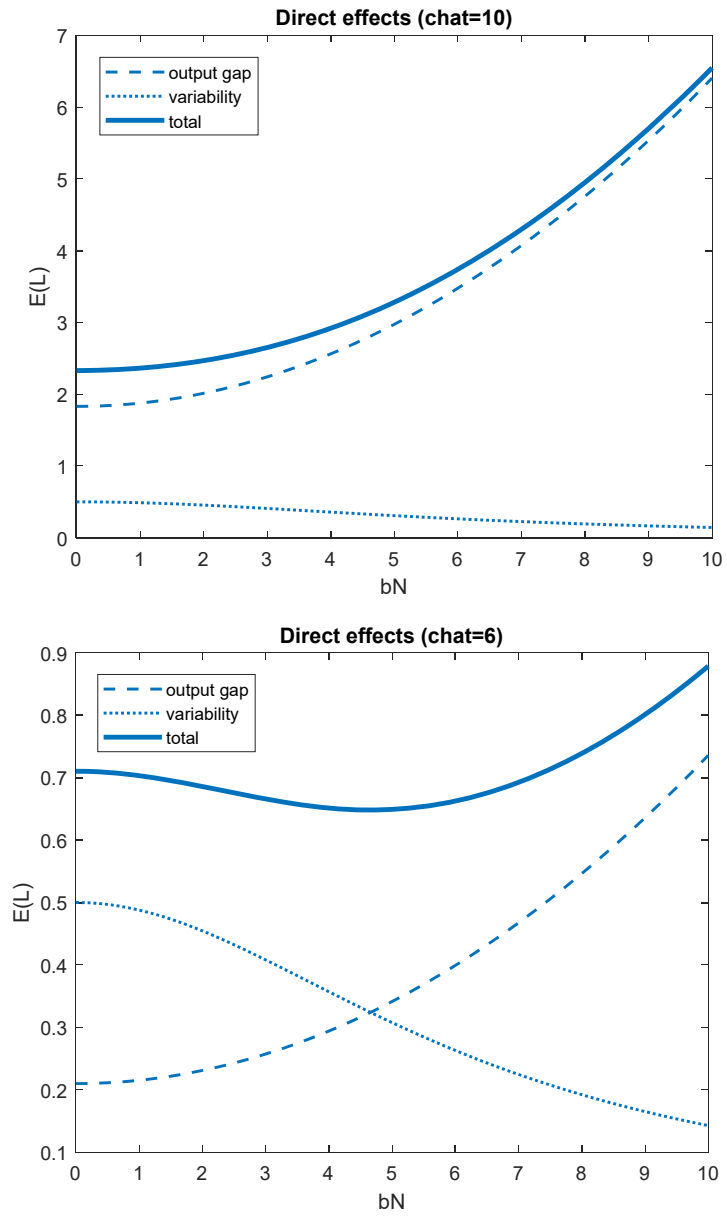
The lower panel of figure 1 does the same for a relatively low value for the consumption target ($\hat{c} = 6$), so that the case in proposition 1b applies. Compared with the upper panel, now the output gap argument has less weight. Hence, the sum of the two effects (the line 'total') is now decreasing for low values of the nominal debt, flat at a value of 4.6 for b_N and increasing beyond. This indicates that the optimal amount of nominal debt is now strictly positive, as reflected in proposition 1b.

These propositions can be understood as follows. Nominal bonds are welfare-increasing as they reduce the impact of supply shocks, whereas they are welfare-reducing as they increase both the impact of destabilizing information errors and the inflation bias of discretionary monetary policies. If the former argument is insufficiently strong, *i.e.* the variance of supply shocks is sufficiently small, then the introduction of an infinitesimally small amount of nominal bonds is welfare-reducing. Then, proposition 1a applies: the optimal amount of nominal debt is zero. On the other hand, if the variance of supply shocks is sufficiently large, then the introduction of an infinitesimally small amount of nominal bonds is welfare-increasing. Now, proposition 1b applies. Increasing the share of nominal bonds further reduces their net welfare gain as the loss due to the inflation bias increases faster than the other two factors (as can be seen from inspecting equation (31)). It is then optimal to increase the nominal debt up to the point as indicated in equation (33), where the net welfare gain of a further increase in the nominal debt is zero or the stock of nominal debt hits its upper bound.⁸

Although intuitive, the results are inconsistent with the model itself, as they treat structural output as exogenous, whereas it is endogenous. Therefore, we now add indirect effects and see what that implies for the results.

⁸The result that is reflected in propositions 1a and 1b are close to those in Calvo and Guidotti (1990). They also find that the optimal amount of nominal debt is increasing in the variance of shocks and decreasing in the inflation bias, but only after some threshold point has been crossed.

Figure 1: Welfare loss: direct effects



4.2 Direct and indirect effects

Different from the previous subsection, we now differentiate $E(L)$ in equation (30) with respect to b_N , taking account of the endogeneity of structural output,

$$\begin{aligned}\frac{dE(L)}{db_N} &= \frac{\partial E(L)}{\partial b_N} + \frac{dE(L)}{dy'_n} \frac{\partial y'_n}{\partial b_N} \\ &= (\delta\epsilon)^2 b_N \left[(\hat{c} - y'_n)^2 - \frac{\theta Var(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \right] \\ &\quad + \frac{2(1 - \delta(\epsilon b_N)^2)(\delta\epsilon)^2 b_N \theta Var(\mu)(\hat{c} - y'_n)}{(1 + \delta(\epsilon b_N)^2)^2 D}\end{aligned}\tag{34}$$

where D is a shortcut for

$$\sqrt{x^2 + \frac{4\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \theta Var(\mu)}$$

The first line at the RHS of equation (34) recalls the direct effects in equation (31); the second line represents the indirect effects. There are two kinds of indirect effects. The first is that an increase in nominal debt changes debt service costs and, through a change of taxation, the levels of structural output and average consumption. Secondly, through changing structural output, the increase in nominal debt changes the output gap which is the source of the inflation bias. The term on the last line of equation (34) takes the two indirect effects together, since they are proportional with each other.

One can see immediately from equation (34) that $b_N = 0$ obeys the first-order condition. Appendix D elaborates the second derivative of $E(L)$ with respect to b_N and derives that at the point $b_N = 0$ this derivative equals $(\delta\epsilon)^2(x^2 + \theta Var(\mu))$, which is strictly positive. Furthermore, appendix D derives that $dE(L)/db_N > 0$ for all $b_N > 0$. Hence, the optimal financing structure of public debt consists of price-indexed debt only,

$$b_N^* = 0\tag{35}$$

We state this result in proposition 2.

Proposition 2:

If we account for the endogeneity of structural output, the optimal financing structure of the public debt features zero nominal debt.

In order to explain this result, it must be the case that the indirect effects imply a welfare loss for small positive values of b_N , as proposition 2 holds irrespective the sign of $(\hat{c} - y'_n) - \sqrt{\theta Var(\mu)}$. This can be easily verified. The two indirect effects turn out to be proportional with the derivative of debt service costs with respect to the stock of nominal debt (see equation (27)). This derivative is positive for small positive values of b_N , as discussed above.

To show that the indirect effects are proportional with debt service costs, we rewrite $(dE(L)/dy'_n)(dy'_n/db_N)$ in equation (34) as follows:

$$\begin{aligned}
& \frac{dE(L)}{dy'_n} \frac{dy'_n}{db_N} \\
&= \left(\frac{\partial E(L)}{\partial \tilde{p}} \frac{d\tilde{p}}{dy'_n} + \frac{\partial E(L)}{\partial y'_n} \right) \left(\frac{dy'_n}{d(r_R b + P b_N)} \right) \left(\frac{d(r_R b + P b_N)}{db_N} \right) \\
&= \left(-\delta(1 + \delta(\epsilon b_N)^2)(\hat{c} - y'_n) \right) (-\epsilon) \left(\frac{d(r_R b + P b_N)}{db_N} \right) \quad (36)
\end{aligned}$$

Note that we have split $dE(L)/dy'_n$ in equation (34) into two terms in order to show that there are two indirect effects, one that runs through average inflation and another that runs through average consumption (structural output). Because they are proportional to each other, they show up as one term in expression (34).

Above, we have shown that the derivative of debt service costs with respect to the stock of nominal debt, $d(r_R b + P b_N)/db_N$, is negative for large values of nominal debt, $b_N > 1/(\sqrt{\delta}\epsilon)$. This does not imply that the total welfare effect changes sign, however. We recall our finding in the discussion of direct effects that the welfare gain from better stabilization properties becomes smaller, the higher becomes the amount of nominal debt.

Figure 2 illustrates. The upper panel assumes $\hat{c} = 10$ and is closely connected to the upper panel of figure 1. Indeed, the line 'variability' which refers to a direct effect, is the same in the two panels. The lines 'output gap' and 'total' in figure 2 look very similar to the corresponding lines in figure 1. Actually, they increase somewhat more than in figure 1. This does not change the conclusion we might have drawn from figure 1: for a sufficiently high value for \hat{c} , the optimal stock of nominal debt equals zero. Accounting for indirect effects does not change this conclusion.

More interesting is the case of $\hat{c} = 6$, as reflected in the lower panel of figure 2. Again, the line 'variability' mimics the corresponding line in figure 1, whereas the line 'output gap' in figure 2 increases a little more over the

range $((0, b)$ than in figure 1. This subtle difference is enough to change the nature of the line 'total' however. In figure 2, more nominal debt produces a higher welfare loss if we account for indirect effects, reflecting our result in proposition 2.

One may wonder why it is that the welfare gains from better stabilization properties play a subordinate role. Indeed, one could argue that if the variance of supply shocks is sufficiently large, nominal bonds should outperform indexed bonds. The reason that this is not the case is that a high variance of supply shocks produces also a high covariance between output and inflation and thus a high risk premium on nominal bonds. A large variance of supply shocks does not only raise the marginal welfare gain from better stabilization policies, it also increases the marginal welfare loss from higher debt service costs.

A similar argument holds true for the variance of information errors. One could argue that a sufficiently low variance of information errors would boost the gains from better stabilization policies and would make a marginal increase in nominal debt welfare-increasing. This is not so, even if $Var(\eta)$ takes its minimum value of zero. The reason is, again, that a low variance of information errors corresponds to a high output-inflation covariance and thus a huge risk premium.

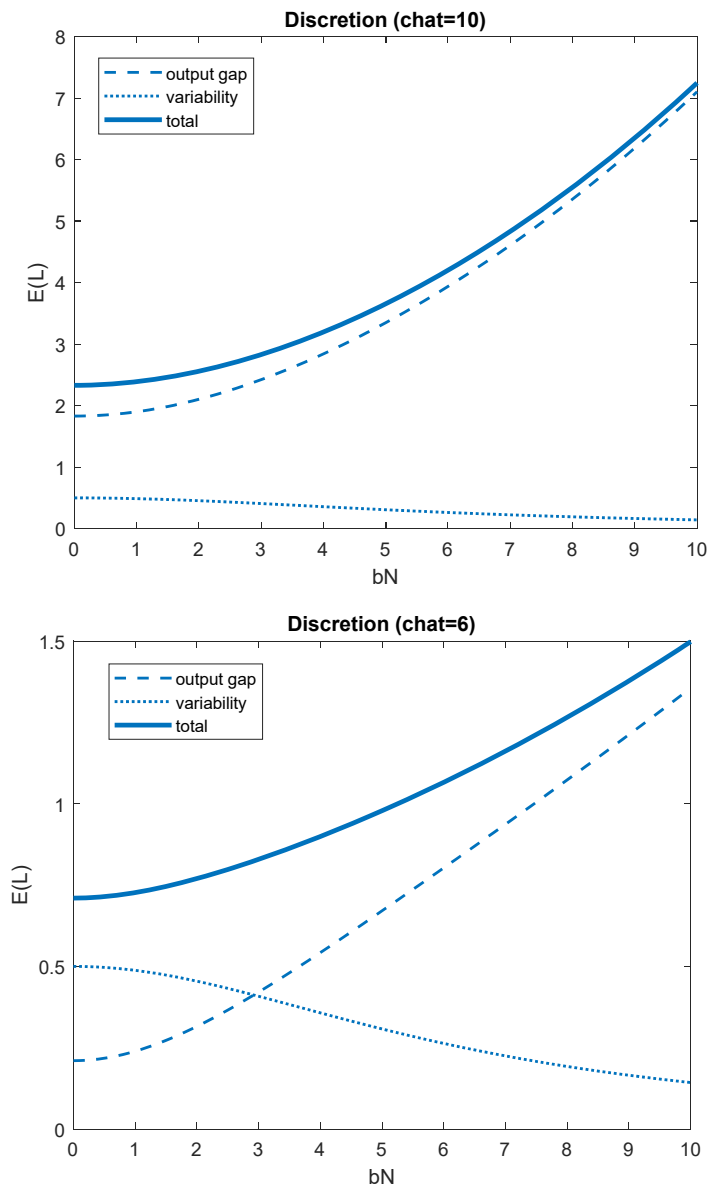
Reducing the target rate for consumption cannot reverse the sign of the marginal welfare effect either. Although this reduces the welfare costs of nominal bonds that relate to the inflation bias, it also increases the inflation risk premium and thus the welfare costs associated with higher debt service costs.

5 The role of discretion

Until now, we have assumed that monetary policies are discretionary in nature. In other words, the central banker in our model lacks the technology to commit himself to optimal policies and to not give in to the temptation to produce surprise inflation after expectations by the public have been formed (and have affected nominal interest rates). Many adhere to the concept of discretionary policies, probably because these policies are time-consistent and there is empirical evidence supporting the concept.⁹ It cannot be denied that the concept is debatable, however. Indeed, one can think of many reasons why a central banker would want to preserve the reputation

⁹For the US, Ireland (1999) and Ruge-Murcia (2003) present empirical evidence for an inflation bias, whereas Surico (2008) finds that this bias has disappeared over time.

Figure 2: Welfare loss: total effects



of a dependable policymaker (Barro and Gordon, 1983b).

In this section, we will therefore explore two alternatives. One is that of committed monetary policies, *i.e.* policies of a central banker who does not give in to the temptation to inflate in order to reduce the output gap. Like the discretionary central banker, the committed central banker does use inflation to stabilize the economy, however. In a sense, committed monetary policies can then be said to be optimal (Alesina and Grilli, 1992): they stabilize the economy without producing unnecessarily high inflation as is the case with discretionary monetary policies.

The second alternative concept is that of monetary policies under imperfect credibility. As that concept builds upon the concept of commitment, we start with the concept of commitment.

5.1 The case of monetary policies under commitment

In the case of commitment, the central banker has access to a commitment technology. Hence, the banker does not give in to the temptation to produce inflation, although that would be optimal after expectations have been formed. On the other hand, the central banker will use the inflation instrument in order to smooth negative or positive changes in output, just as he does in the discretionary case. Similarly, like before, the central banker cannot observe output shocks precisely and has to act on the basis of an output signal only.¹⁰ Taking things together, this means that the equation for price inflation differs only in one respect from the one we derived for the case of discretion (equation (14)), namely the inflation bias:¹¹

$$\tilde{p} = - \left(\frac{\delta \epsilon b_N}{1 + \delta (\epsilon b_N)^2} \right) \theta (\mu + \eta) \quad (37)$$

The equation of consumption is identical to the one we derived for the case of discretion (equation (17)).

¹⁰This interpretation of commitment follows that in Alesina and Grilli (1992). Note that it is different from the case of rules (Kydland and Prescott, 1977; Barro and Gordon, 1983b; Athey *et al.*, 2005), in which case policies are unresponsive to output shocks. For the purpose of this paper, the case of rules is not very interesting. Nominal debt would not play any role as it would not affect output variability, expected inflation or the inflation risk premium.

¹¹Similar to the case of discretionary policies, equation (37) generalizes the equation in Alesina and Grilli (1992). Their equation is obtained if we add to their model imperfect information on the part of the central banker and further allow the coefficient of the Phillips curve to deviate from one.

The joint inflation-consumption distribution changes also in only one respect: expected inflation. Obviously, expected inflation is zero in the case of commitment:

$$E(\tilde{p}) = 0 \quad (38)$$

Expected consumption, the variance of inflation, the variance of consumption and the covariance between inflation and consumption do not change; the expressions for these variables are identical to their counterparts in the case of discretion (equations (17) to (20)).

Compared to the case of discretion, the welfare analysis simplifies, as the term that relates to the inflation bias drops out. Different from the case of discretion, nominal debt does not lower welfare by fueling the inflation bias directly and indirectly through structural output in the case of commitment. This can be seen in the following expressions for the social welfare loss and the corresponding derivative with respect to nominal debt:

$$E(L) = \frac{1}{2}\delta(\hat{c} - y'_n)^2 + \frac{1}{2}\delta \left(\frac{1 + (1 - \theta)\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) Var(\mu) \quad (39)$$

$$\begin{aligned} \frac{dE(L)}{db_N} &= \frac{\partial E(L)}{\partial b_N} + \frac{dE(L)}{dy'_n} \frac{\partial y'_n}{\partial b_N} \\ &= -(\delta\epsilon)^2 b_N \frac{\theta Var(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \\ &\quad + \frac{2(1 - \delta(\epsilon b_N)^2)(\delta\epsilon)^2 b_N \theta Var(\mu)(\hat{c} - y'_n)}{(1 + \delta(\epsilon b_N)^2)^3 D} \end{aligned} \quad (40)$$

Looking at the derivative in equation (40), we see that, like in the case of discretion, $b_N = 0$ solves the first-order condition $dE(L)/db_N = 0$. Furthermore, appendix E derives that at $b_N = 0$, $d^2E(L)/(db_N)^2 = (\delta\epsilon)^2\theta Var(\mu) > 0$. $b_N = 0$ thus represents a local optimum. However, the expression for the derivative in the case of commitment differs fundamentally from its counterpart in the case of discretion. Appendix E show that $dE(L)/db_N$ turns negative beyond a threshold value for b_N . This implies that $b_N = 0$ does not need to a global optimum; the optimum is either $b_N = 0$ or $b_N = b$.¹²

Appendix E elaborates expressions for $E(L)$ when $b_N = 0$ and when $b_N = b$ and uses these to derive that there is a critical value for b , to be

¹²This result is reminiscent of the result that Calvo and Guidotti (1990) derive for the case of commitment: finance all the public debt with nominal bonds.

denoted \tilde{b} , that defines which of the two solutions applies: if $b < \tilde{b}$, $b_N = 0$ is optimal and if $b > \tilde{b}$, $b_N = b$ is optimal. In the borderline case $b = \tilde{b}$, an indifference result emerges: the options of financing the public debt fully with nominal bonds or fully with price-indexed bonds yield identical levels of social welfare.

We state the result on the optimal debt structure in proposition 3.

Proposition 3:

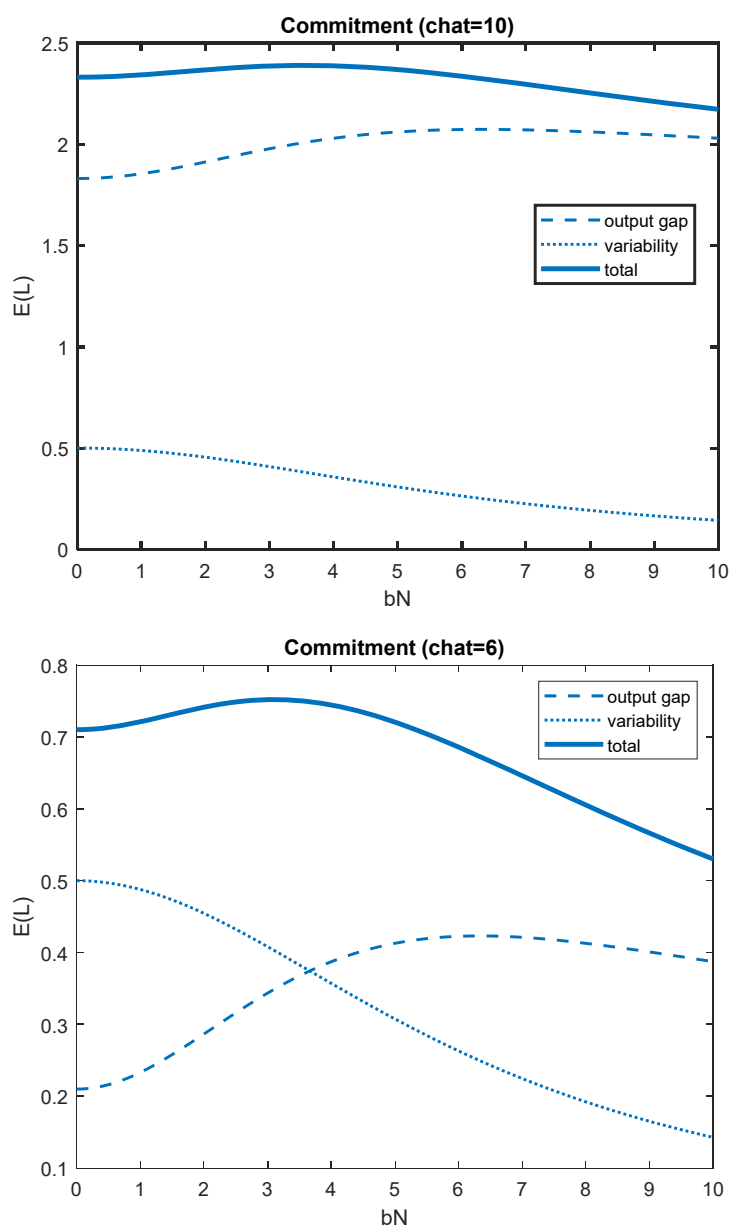
In case monetary policies are conducted under commitment, the optimal financing structure of the public debt features either zero nominal debt or zero inflation-linked debt.

Figure 3 illustrates proposition 3 for the same parameter configurations as used before. The lines labelled 'variability' in the two panels correspond to their counterparts in figures 1 and 2. The lines labelled 'output gap' are different, however. In figure 3, they are decreasing beyond a critical value for b_N . This explains that the lines labelled 'total' in the two panels are hump-shaped. In both cases, $b_N = b$ implies a lower welfare loss than $b_N = 0$. The contrast with the case of discretion cannot be more clear.

5.2 A more general case of monetary policies

Given the difference in results for the case of discretion and commitment, it is interesting to study also a more general case that encompasses these two cases. We adopt an approach based on Cukierman and Liviatan (1991). They develop a model for a non-stochastic world based on two assumptions. One is that the policymaker can be weak (W) or strong (S). A weak policymaker is a policymaker who is unable to commit to its earlier announcement; a strong policymaker always sticks to his earlier announcement. The probability that the policymaker is strong (weak) is α ($1 - \alpha$). The other assumption is that the public is imperfectly informed about the type of policymaker; only the probabilities α and $1 - \alpha$ are known. Cukierman and Liviatan derive that it is optimal for the strong policymaker not to announce a zero rate of inflation, but a rate of inflation equal to $(1 - \alpha)\tilde{p}_D$, where \tilde{p}_D is the rate of inflation chosen by the weak policymaker. The reason is that the public accounts for the possibility that the policymaker in office is weak and will produce the discretionary rate of inflation. This raises their inflation expectations (compared to the perfect information case), which in turn induces the strong policymaker to announce (and produce) the rate of inflation $(1 - \alpha)\tilde{p}_D$.

Figure 3: Welfare loss: the case of commitment



We use these results to distinguish between a strong and weak policymaker who differ in their responsiveness to the output gap. Similar to the case of commitment studied above, we assume the strong and weak policymaker to respond in an identical way to (imperfectly observed) output shocks.

This then yields that the rate of price inflation chosen by the weak policymaker mimics the rate elaborated above for the case of discretion (equation (14)):

$$\tilde{p}_W = \delta \epsilon b_N (\hat{c} - y'_n) - \left(\frac{\delta \epsilon b_N}{1 + \delta (\epsilon b_N)^2} \right) \theta (\mu + \eta) \quad (41)$$

The strong policymaker produces the following rate of price inflation:

$$\tilde{p}_S = (1 - \alpha) \delta \epsilon b_N (\hat{c} - y'_n) - \left(\frac{\delta \epsilon b_N}{1 + \delta (\epsilon b_N)^2} \right) \theta (\mu + \eta) \quad (42)$$

Note that this rate of price inflation is always $(1 - \alpha) \delta \epsilon b_N (\hat{c} - y'_n)$ higher than in case of commitment (equation (37)).

The expected rate of inflation averages the rates of inflation chosen by the two types of policymakers:

$$E(\tilde{p}) = (1 - \alpha) \tilde{p}_W + \alpha \tilde{p}_S = (1 - \alpha^2) \delta \epsilon b_N (\hat{c} - y'_n) \quad (43)$$

Expected consumption can be derived to be identical to the expression that applies in the cases of discretion and commitment. The same holds true for the variances of inflation and consumption and their covariance. The case studied in this subsection thus generalizes the two earlier cases and includes them as special cases: if $\alpha = 0$, we are back in the world of discretionary policies (equation (16)); if $\alpha = 1$, the commitment case applies (equation (37)).

The expression for social welfare also generalizes the two earlier expressions in equations (30) and (39):

$$\begin{aligned} E(L) &= \frac{1}{2} \delta (1 + \delta (1 - \alpha^2)^2 (\epsilon b_N)^2) (\hat{c} - y'_n)^2 \\ &+ \frac{1}{2} \delta \left(\frac{1 + (1 - \theta) \delta (\epsilon b_N)^2}{1 + \delta (\epsilon b_N)^2} \right) Var(\mu) \end{aligned} \quad (44)$$

The same holds true for the expression for $dE(L)/db_N$:

$$\begin{aligned}
\frac{dE(L)}{db_N} &= (1 - \alpha^2)^2 (\delta\epsilon)^2 b_N (\hat{c} - y'_n)^2 \\
&+ (1 - \alpha^2)^2 \left(\frac{2(\delta\epsilon)^2 b_N (\hat{c} - y'_n) \delta(\epsilon b_N)^2 \theta \text{Var}(\mu) (1 - \delta(\epsilon b_N)^2)}{(1 + \delta(\epsilon b_N)^2)^3 D} \right) \\
&+ \left(\frac{2(\delta\epsilon)^2 b_N (\hat{c} - y'_n) \theta \text{Var}(\mu) (1 - \delta(\epsilon b_N)^2)}{(1 + \delta(\epsilon b_N)^2)^3 D} \right) \\
&- \frac{(\delta\epsilon)^2 b_N \theta \text{Var}(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \\
&= (1 - \alpha^2)^2 \left(\frac{dE(L)}{db_N} \Big|_D \right) + (1 - (1 - \alpha^2)^2) \left(\frac{dE(L)}{db_N} \Big|_C \right)
\end{aligned} \tag{45}$$

The last line of this equation shows that the derivative $dE(L)/db_N$ in the general case can be written as a weighted average of the corresponding expressions for the case of discretion and commitment ($dE(L)/db_N|_D$ refers to the case of discretion, equation (34), and $dE(L)/db_N|_C$ refers to the case of commitment, equation (40)).

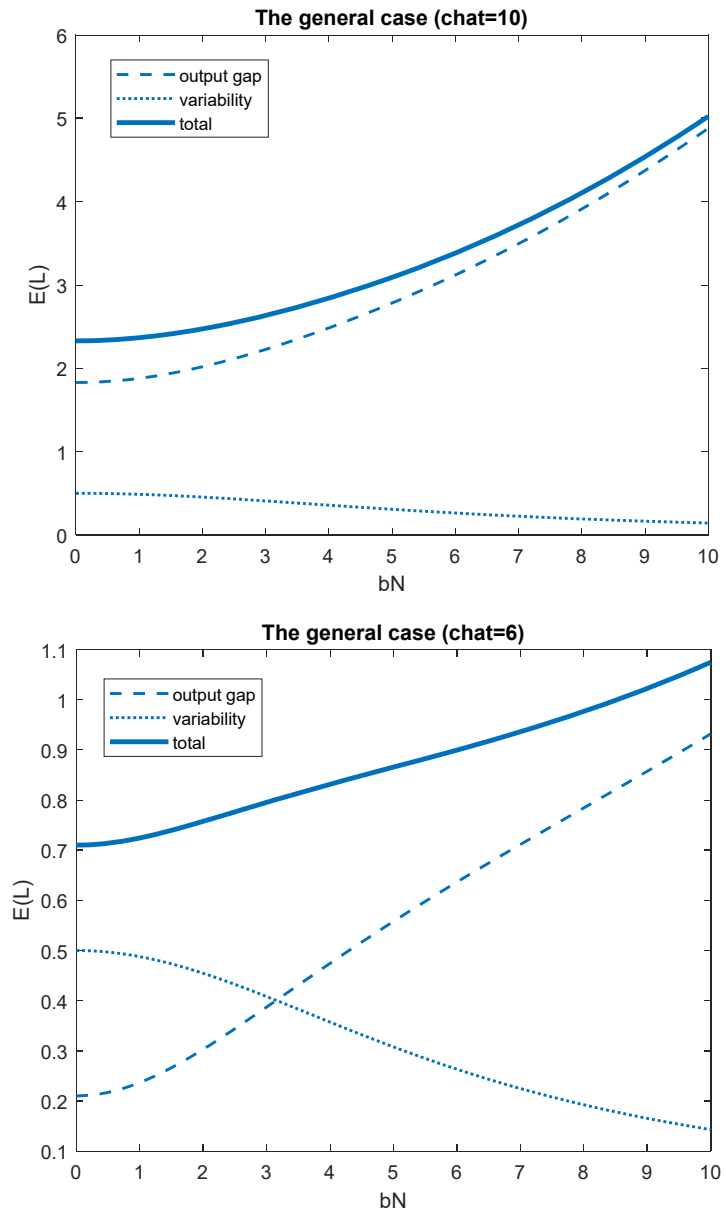
Simulations will then reflect the case of either discretion or commitment. If we take α sufficiently low, the general case will be close to the case of discretion and the optimum amount of nominal bonds will be zero. If, on the other hand, we take α sufficiently close to one and b sufficiently large, the optimal debt structure will involve zero inflation-linked bonds.

Figure 4 illustrates, again using the same parameter configurations as used before and assuming $\alpha = 0.5$. In both panels, the case of discretion dominates that of commitment. Hence, in both panels the optimal amount of nominal debt equals zero. Actually, it takes a quite high value for α to overturn this result. Simulations (not shown for brevity) show that only if the probability that the central banker is a strong policymaker is higher than 75 percent (if $\hat{c} = 6.0$) or higher than 90 percent (when $\hat{c} = 10.0$), it is optimal to finance the public debt entirely with nominal bonds.

6 Conclusions

Let us go back now to where we started. That is the question why governments across the world use mainly nominal bonds to finance their public debts? Constructing a model of discretionary monetary policies and focussing on the direct welfare effects, it seems that we find an answer to that

Figure 4: Welfare loss: the more general case



question. Nominal debt makes monetary policies that are aimed at the stabilization of the economy more effective. This even holds true when output shocks are only imperfectly observed by the central banker. But this analysis that focuses on direct effects only also shows that this result does not generally hold true. If the output gap is sufficiently large or the variance of output shocks sufficiently small, issuing only a small amount of nominal bonds will be welfare-reducing and it is better not to use nominal bonds at all.

More importantly, these results cannot be more than illustrative however, as they are based on an inconsistent approach. To make the approach consistent, we have to account for indirect effects as well. Doing so destroys the results achieved before. Now, the optimal amount of nominal bonds turns out to be zero, irrespective the values of output gap and variance of output shocks. Intuitively, if nominal debt makes monetary policies more effective in stabilizing the economy, this show up in an inflation risk premium, which makes it more costly to use nominal bonds to finance the public debt.

One way to get rid of these results is to assume that monetary policymakers can commit themselves to optimal policies, *i.e.* to assume that policymakers are able not to give in to the temptation to inflate once price expectations (and nominal interest rates) have been set. The assumption of commitment removes the inflation bias from the model and lowers the welfare cost of the inflation risk premium. Assuming commitment, the optimal fraction of the public debt that should be financed with nominal bonds is now 0 or 100 percent. Which of the two policies is optimal, depends again on the value of output gap and variance of output shocks. However, the assumption of commitment does not explain either why governments *mainly* use nominal bonds to finance part of their debts.

Finally, we adopt a more general approach that encompasses the cases of discretion and commitment. Giving equal weights to the two cases, the welfare losses of the nominal bonds under discretion turn out to dominate the gains under commitment and the optimal amount of nominal bonds is zero. Only if the case of commitment is given very large weight, does the opposite result (use only nominal bonds to finance the public debt) emerge.

Overall, our analysis indicates that generally price-index bonds dominate nominal bonds in welfare terms. Returning to the question in the introduction of this paper, our analysis does not give an answer to it. The dominance of nominal bonds must then be attributed to other factors. These may include the liquidity argument or the existence of nominal contracts (see the introduction of this paper). Another argument is that too much indexation may lead people to fear an increase of inflation. Indeed, Fischer and Sum-

mers (1989) and Ball and Cecchetti (1991) argue that indexation reduce the costs of inflation and may thereby give rise to higher inflation. Further research is clearly warranted to answer the unresolved puzzle.

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Appendices

A Linearizing the output equation

In the main text, we have stated the production function and the demand for labour that can be derived from it. For convenience, we repeat them here:

$$y = l^\eta + \mu \quad (\text{A.1})$$

$$l = \left(\frac{w + \tau}{\eta} \right)^{\frac{1}{\eta-1}} \quad (\text{A.2})$$

Upon substituting equation (A.2) into equation (A.1), we derive an equation for output which is non-linear in terms of the tax rate τ :

$$y = \left(\frac{w + \tau}{\eta} \right)^{\frac{\eta}{\eta-1}} + \mu \quad (\text{A.3})$$

Linearizing the first term at the RHS of equation (A.3) around $\tau = 0, \mu = 0$ gives the following equation for output,

$$y = y_0 - \frac{1}{1-\eta} y_0^\eta \tau + \mu \quad (\text{A.4})$$

where y_0 is defined as $(w/\eta)^{(\eta/(\eta-1))}$.

In a similar way, we linearize the equation for tax revenues, $t \equiv \tau l$, in terms of the tax rate,

$$t = l_0(1 + \sigma_l)\tau \quad (\text{A.5})$$

where $l_0 = (w/\eta)^{(1/(\eta-1))}$.

σ_l is the elasticity of labour demand with respect to the tax rate, *i.e.* $\sigma_l \equiv (dl/d\tau)(\tau/l)$, evaluated in $\tau = 0, \mu = 0$. $\sigma_l < 0$, given our concave production function. We assume in addition that $\sigma_l > -1$ in order to ensure that tax revenues are increasing in the tax rate.

Combining equations (A.4) and (A.5) yields an equation for output that is linear in tax revenues,

$$y = y_n - \epsilon t + \mu \quad (\text{A.6})$$

where ϵ is a shortcut for $1/((1-\eta)(1+\sigma_l))$ and y_n represents y_0 . As $0 < \eta < 1$ and $-1 < \sigma_l < 0$, $\epsilon > 0$.

B Deriving expressions for the variances of inflation and consumption and their covariance

Recall the expressions for inflation and consumption from the main text (equations (14) and (15)):

$$\tilde{p} = \delta(\epsilon b_N)(\hat{c} - y'_n) - \left(\frac{\theta \delta(\epsilon b_N)}{1 + \delta(\epsilon b_N)^2} \right) (\mu + \eta) \quad (\text{B.1})$$

$$c = y'_n + \left(\frac{1 + (1 - \theta)\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \mu - \left(\frac{\theta \delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \eta \quad (\text{B.2})$$

Use the equation for inflation to find an expression for the variance of inflation (recall that μ and η are uncorrelated). This gives the first line in equation (B.3). Elaboration yields the last line of equation (B.3), which is equation (18) in the main text.

$$\begin{aligned} \text{Var}(\tilde{p}) &= \left(\frac{\theta \delta(\epsilon b_N)}{1 + \delta(\epsilon b_N)^2} \right)^2 (\text{Var}(\mu) + \text{Var}(\eta)) \\ &= \left(\frac{\delta(\epsilon b_N)}{1 + \delta(\epsilon b_N)^2} \right)^2 \frac{(\text{Var}(\mu))^2}{\text{Var}(\mu) + \text{Var}(\eta)} \\ &= \left(\frac{\delta(\epsilon b_N)}{1 + \delta(\epsilon b_N)^2} \right)^2 \theta \text{Var}(\mu) \end{aligned} \quad (\text{B.3})$$

Following the same procedure gives equations (19) and (20) in the main text.

$$\begin{aligned} \text{Var}(c) &= \left(\frac{1 + (1 - \theta)\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right)^2 \text{Var}(\mu) \\ &+ \left(\frac{\theta \delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right)^2 \text{Var}(\eta) \\ &= \left(\frac{1 + 2(1 - \theta)\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2} \right) \text{Var}(\mu) \\ &+ \left(\frac{\delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right)^2 ((1 - \theta)^2 \text{Var}(\mu) + \theta^2 \text{Var}(\eta)) \\ &= \left(\frac{1 + 2(1 - \theta)\delta(\epsilon b_N)^2 + (1 - \theta)\delta^2(\epsilon b_N)^4}{(1 + \delta(\epsilon b_N)^2)^2} \right) \text{Var}(\mu) \end{aligned} \quad (\text{B.4})$$

$$\begin{aligned}
Cov(c, \tilde{p}) &= \frac{-\theta\delta(\epsilon b_N)(1 + (1 - \theta)\delta(\epsilon b_N)^2)}{(1 + \delta(\epsilon b_N)^2)^2} Var(\mu) \\
&+ \frac{\theta^2\delta^2(\epsilon b_N)^3}{(1 + \delta(\epsilon b_N)^2)^2} Var(\eta) \\
&= \frac{-\theta\delta(\epsilon b_N)}{(1 + \delta(\epsilon b_N)^2)^2} Var(\mu) \\
&+ \frac{\theta\delta^2(\epsilon b_N)^3}{(1 + \delta(\epsilon b_N)^2)^2} (-(1 - \theta)Var(\mu) + \theta Var(\eta)) \\
&= \frac{-\theta\delta(\epsilon b_N)}{(1 + \delta(\epsilon b_N)^2)^2} Var(\mu)
\end{aligned} \tag{B.5}$$

C When do propositions 1a and 1b apply?

Consider the inequality condition in proposition 1a:

$$(\hat{c} - y'_n) \geq \sqrt{\theta Var(\mu)} \tag{C.1}$$

Using the definitions for $y'_n \equiv y_n - \epsilon(r_R b + P b_N)$ and $x \equiv \hat{c} - y_n + \epsilon r_R b$, we rewrite this condition as

$$x + \epsilon P b_N \geq \sqrt{\theta Var(\mu)} \tag{C.2}$$

Using the structural equation for P , equation (25), we rewrite the inequality condition in the following form:

$$\frac{1}{2}x + \frac{1}{2}\sqrt{x^2 + \frac{4\delta(\epsilon b_N)^2\theta Var(\mu)}{(1 + \delta(\epsilon b_N)^2)^2}} \geq \sqrt{\theta Var(\mu)} \tag{C.3}$$

Defining z as $x/\sqrt{\theta Var(\mu)}$, we rewrite this condition as follows:

$$\frac{1}{2}z + \frac{1}{2}\sqrt{z^2 + \frac{4\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2}} \geq 1 \tag{C.4}$$

For the term $\delta(\epsilon b_N)^2/(1 + \delta(\epsilon b_N)^2)^2$, we calculate the following derivative with respect to b_N :

$$d\left(\frac{\delta(\epsilon b_N)^2}{(1 + \delta(\epsilon b_N)^2)^2}\right)/db_N = \frac{2\delta\epsilon^2 b_N(1 - \delta(\epsilon b_N)^2)}{(1 + \delta(\epsilon b_N)^2)^3} \tag{C.5}$$

From this expression, we derive that the derivative term $d(\delta(\epsilon b_N)^2/(1 + \delta(\epsilon b_N)^2)^2)/db_N$ is positive if $0 < b_N < 1/(\sqrt{\delta}\epsilon)$, zero if $b_N = 1/(\sqrt{\delta}\epsilon)$ and

negative if $b_N > 1/(\sqrt{\delta}\epsilon)$. Hence, we conclude from this that the term $\delta(\epsilon b_N)^2/(1 + \delta(\epsilon b_N)^2)^2$ has a minimum of zero (for $b_N=0$) and a maximum of $1/4$ (for $b_N = 1/(\sqrt{\delta}\epsilon)$).

In terms of equation (C.4), we derive an inequality condition for z if $\delta(\epsilon b_N)^2/(1 + \delta(\epsilon b_N)^2)^2$ takes its minimum value:

$$\frac{1}{2}z + \frac{1}{2}\sqrt{z^2} \geq 1 \quad z \geq 1 \quad (\text{C.6})$$

Similarly, we derive an inequality condition if $\delta(\epsilon b_N)^2/(1 + \delta(\epsilon b_N)^2)^2$ takes its maximum value:

$$\frac{1}{2}z + \frac{1}{2}\sqrt{z^2 + 1} \geq 1 \quad z \geq 3/4 \quad (\text{C.7})$$

Recalling that $z \equiv x/\sqrt{\theta \text{Var}(\mu)}$, we derive that $x \geq \sqrt{\theta \text{Var}(\mu)}$ is a sufficient condition for $(\hat{c} - y'_n) \geq \sqrt{\theta \text{Var}(\mu)}$ and $x < 3/4\sqrt{\theta \text{Var}(\mu)}$ is a sufficient condition for $(\hat{c} - y'_n) < \sqrt{\theta \text{Var}(\mu)}$.

D Proof of proposition 2

This appendix proves proposition 2 in the main text.

We start to write down the first-order condition for optimal nominal debt, based on equation (34) in the main text:

$$\begin{aligned} \frac{dE(L)}{db_N} &= (\delta\epsilon)^2 b_N \left[(\hat{c} - y'_n)^2 - \frac{\theta \text{Var}(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \right] \\ &+ \frac{2(1 - \delta(\epsilon b_N)^2)(\delta\epsilon)^2 b_N \theta \text{Var}(\mu)(\hat{c} - y'_n)}{(1 + \delta(\epsilon b_N)^2)^2 D} = 0 \end{aligned} \quad (\text{D.1})$$

One easily derives from this expression that $b_N = 0$ solves this first-order condition. In order to find whether this solution represents a minimum or maximum, we elaborate the second-order derivative $d^2E(L)/(db_N)^2$ and evaluate it at $b_N = 0$. This yields the following expression:

$$\frac{d^2E(L)}{(db_N)^2} \Big|_{b_N=0} = (\delta\epsilon)^2(x^2 + \theta \text{Var}(\mu)) > 0 \quad (\text{D.2})$$

This indicates the solution $b_N = 0$ reflects a local minimum.

In order to find out whether equation (D.1) has more solutions, we evaluate $dE(L)/db_N$ for $b_N > 0$. Before doing that however, we rewrite equation (D.1) in a more convenient form. Using the definition of D as given in the

main text, we rewrite y'_n as $\hat{c} - 1/2(D + x)$ and thus $\hat{c} - y'_n$ as $1/2(D + x)$. Hence, we can write equation (D.1) as follows:

$$\begin{aligned} \frac{dE(L)}{db_N} &= (\delta\epsilon)^2 b_N \frac{1}{4} (D + x)^2 \\ &- \frac{(\delta\epsilon)^2 b_N \theta \text{Var}(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \left(1 - \frac{(D + x)}{D} (1 - \delta(\epsilon b_N)^2) \right) = 0 \end{aligned} \quad (\text{D.3})$$

In order to evaluate $dE(L)/db_N$ for any $x > 0$, we proceed in two steps. First, we evaluate $dE(L)/db_N$ in case $x = 0$. This yields $dE(L)/db_N|_{x=0} = 0$ (use that in case $x = 0$, D^2 reads as $4\theta\delta(\epsilon b_N)^2 \text{Var}(\mu)/(1 + \delta(\epsilon b_N)^2)^2$).

Second, we differentiate $dE(L)/db_N$ with respect to x . This yields the following expression (use that $dD/dx = x/D$ and that $(D^2 - x^2)/(4\delta(\epsilon b_N)^2) = \theta \text{Var}(\mu)/(1 + \delta(\epsilon b_N)^2)^2$):

$$\frac{d^2 E(L)}{db_N dx} = (\delta\epsilon)^2 b_N \frac{(D + x)^2}{D} \left(\frac{1}{2} + \left(\frac{D - x}{D} \right)^2 \left(\frac{1}{4\delta(\epsilon b_N)^2} - \frac{1}{4} \right) \right) \quad (\text{D.4})$$

We derive that for all $x \geq 0$, $d^2 E(L)/(db_N dx) = 0$ if $b_N = 0$. This does not follow immediately from evaluating $d^2 E(L)/(db_N dx)$, as this gives an indeterminate outcome for $b_N = 0$. Applying the rule of l'Hôpital, we derive $\lim_{b_N \rightarrow 0} d^2 E(L)/(db_N dx)$ by elaborating $\lim_{b_N \rightarrow 0} f'(b_N)/(\lim_{b_N \rightarrow 0} g'(b_N))$, where $f(b_N) = (1 - x/D)^2$ and $g(b_N) = 4\delta\epsilon^2 b_N$.

Next, we derive that for all $x \geq 0$, $d^2 E(L)/(db_N dx) > 0$ if $b_N > 0$.

Combining the results from these two steps, we derive that for all $x > 0$, $dE(L)/(db_N) = 0$ if $b_N = 0$ and $dE(L)/(db_N) > 0$ if $b_N > 0$. Hence, $b_N = 0$ is the only solution to the first-order condition (D.1) for any $x > 0$.

E Proof of proposition 3

In order to find the optimal debt financing structure in the case of commitment, we rewrite equation (40) as follows:

$$\frac{dE(L)}{db_N} = \frac{(\delta\epsilon)^2 b_N \theta \text{Var}(\mu)}{(1 + \delta(\epsilon b_N)^2)^2} \left[-1 + \left(\frac{1 - \delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \left(\frac{D + x}{D} \right) \right] \quad (\text{E.1})$$

This expression clearly shows that $b_N = 0$ solves the first-order condition. Furthermore, we can use it to derive that at $b_N = 0$ $d^2 E(L)/(db_N)^2 = (\delta\epsilon)^2 \theta \text{Var}(\mu) > 0$.

Define $H(b_N)$ as the term between right brackets in equation (E.1):

$$H(b_N) = -1 + \left(\frac{1 - \delta(\epsilon b_N)^2}{1 + \delta(\epsilon b_N)^2} \right) \left(\frac{D + x}{D} \right)$$

Differentiation yields that

$$\frac{dH}{db_N} = \frac{-4\delta\epsilon^2 b_N}{(1 + \delta(\epsilon b_N)^2)^2} \left(\frac{D + x}{D} \right) - \frac{4x\delta\epsilon^2 b_N \theta Var(\mu)(1 - \delta(\epsilon b_N)^2)^2}{D^3(1 + \delta(\epsilon b_N)^2)^4}$$

which is negative for all $b_N > 0$. Noting that $H(0) = 1$ and $H(1/(\sqrt{\delta\epsilon})) = -1$, it must be that the H function switches sign (from positive to negative) at a threshold value, say \hat{b}_N , for which $0 < \hat{b}_N < 1/(\sqrt{\delta\epsilon})$. Given that the H function is negative for some values of b_N , the optimal value of nominal debt is either zero or b .

To see whether $b_N = 0$ or $b_N = b$ corresponds with a lower welfare loss, we elaborate $E(L)$ as specified in equation (39) for these two candidate solutions. This yields the following results:

$$\begin{aligned} E(L)|_{b_N=0} &= \frac{1}{2}\delta x^2 + \frac{1}{2}\delta Var(\mu) \\ E(L)|_{b_N=b} &= \frac{1}{2}\delta \left(\frac{1}{2}x + \frac{1}{2}\sqrt{x^2 + \frac{4\delta(\epsilon b)^2\theta Var(\mu)}{(1 + \delta(\epsilon b)^2)^2}} \right)^2 \\ &\quad + \frac{1}{2}\delta Var(\mu) - \frac{1}{2}\frac{\delta^2(\epsilon b)^2\theta Var(\mu)}{(1 + \delta(\epsilon b)^2)^2} \end{aligned}$$

We can now derive that the sign of $(E(L)|_{b_N=0} - E(L)|_{b_N=b})$ equals the sign of $b(x^2(-1 + \delta(\epsilon b)^2) + \delta^3(\epsilon b)^6\theta Var(\mu)/(1 + \delta(\epsilon b)^2)^2)$. Define this expression as bJ . Differentiation of J with respect to b yields the following result:

$$\frac{\partial J}{\partial b} = 2\delta\epsilon^2 b \left(x^2 + \frac{\delta^2(\epsilon b)^4\theta Var(\mu)(3 + \delta(\epsilon b)^2)}{(1 + \delta(\epsilon b)^2)^3} \right) > 0$$

Note that $J < 0$ for sufficiently small, but positive values of b . Since $\partial J/\partial b > 0$, $J > 0$ for values of $b > \tilde{b}$, where \tilde{b} is the value for b that solves $J = 0$.